Sum and Product

QE11-V1 · storyboard · QE11 Sum and product of the roots of x² + bx + c. Read each picture with its sentences.

Picture 1

The roots are r and s. Expand $(x-r)(x-s)$.

$$x^2 - rx - sx + rs$$
$$= x^2 - (r+s)x + rs$$

For $x^2 + bx + c$**:**

$$r + s = -b, \qquad rs = c$$

1. Suppose the roots are r and s.
2. Expand x minus r, times x minus s.
3. The four products: x squared, minus r x, minus s x, plus r s.
4. Take out x from the middle terms: x squared, minus r plus s times x, plus r s.
5. So for x squared plus b x plus c, the sum of the roots is negative b, and the product is c.

Picture 2

The sum and the product of the roots of $x^2 + 15x - 324 = 0$.

$$\boxed{r + s = -15, \qquad rs = -324}$$ no need to solve

Check: $12 + (-27) = -15, \quad 12 \times (-27) = -324$ ✓

6. Find the sum and the product of the roots of x squared, plus 15 x, minus 324, equals 0.
7. There is no need to solve: the sum is negative 15, and the product is negative 324.
8. The roots are 12 and negative 27, and they check.

Picture 3

The roots of $x^2 + bx + c$ have sum -3 and product -40. Find them.

$$x^2 + 3x - 40 = 0 \quad b = -(\text{sum}),\ c = \text{product}$$

$$(x + 8)(x - 5) = 0$$

$$\boxed{x = -8 \quad \text{or} \quad x = 5}$$

9. The roots of x squared plus b x plus c have sum negative 3 and product negative 40.
10. Find them.
11. b is the opposite of the sum, so b is 3.
12. c is the product, negative 40.
13. Factor: product negative 40, sum 3: 8 and negative 5.
14. The roots are negative 8 and 5.

Picture 4

Trap $x^2 + 15x - 324$

$\times\ r + s = 15$

$\checkmark\ \ r + s = -15$

15. Watch for the trap: the sum is not 15.
16. The sum is negative b: negative 15.

Picture 5

Trap $\text{roots } -3 \text{ and } 7$

$\times\ (x - 3)(x + 7)$

$\checkmark\ \ (x + 3)(x - 7)$

17. Another trap: roots negative 3 and 7 do not give x minus 3, times x plus 7.
18. The root r comes from x minus r: x plus 3, times x minus 7.