Without Finding the Roots

QE13-V1 · storyboard · QE13 Use the roots without finding them. Read each picture with its sentences.

Picture 1

r and s are the roots of $x^2 - 7x + 3 = 0$. Find $(r-1)(s-1)$.

$$(r-1)(s-1) = rs - r - s + 1 \quad \text{expand}$$

$$= rs - (r+s) + 1$$

$$r + s = 7, \quad rs = 3 \quad -b \text{ and } c$$

$$= \boxed{3 - 7 + 1 = -3}$$

1. r and s are the roots of x squared, minus 7 x, plus 3, equals 0.
2. Find r minus 1, times s minus 1.
3. Expand: r s, minus r, minus s, plus 1.
4. That uses only the sum and the product: r s, minus r plus s, plus 1.
5. Here the sum is 7, and the product is 3.
6. So the answer is 3 minus 7 plus 1, which is negative 3.

Picture 2

r and s are the roots of $2x^2 - 6x + 1 = 0$. Find $\frac{1}{r} + \frac{1}{s}$.

$$\frac{1}{r} + \frac{1}{s} = \frac{r+s}{rs}$$ a common denominator

$$r + s = 3, \quad rs = \frac{1}{2}$$ $-\frac{b}{a}$ and $\frac{c}{a}$

$$= \boxed{\frac{3}{1/2} = 6}$$

7. r and s are the roots of 2 x squared, minus 6 x, plus 1, equals 0.
8. Find 1 over r, plus 1 over s.
9. Use a common denominator: r plus s, over r s.
10. The sum is 6 over 2, which is 3.
11. The product is 1 over 2.
12. So the answer is 3 divided by one half, which is 6.

Picture 3

Trap $(r-1)(s-1)$

$\times\ rs+1$

$\checkmark\ \ rs-(r+s)+1$

13. Watch for the trap: r minus 1, times s minus 1, is not r s plus 1.
14. Don't forget the cross products, minus r and minus s.

Picture 4

Trap $\frac{1}{r}+\frac{1}{s}$

$\times\ \frac{1}{r+s}$

$\checkmark\ \ \frac{r+s}{rs}$

15. Another trap: 1 over r, plus 1 over s, is not 1 over r plus s.
16. Use a common denominator: r plus s, over r s.